Augmenting Grid-Based Contours to Improve Thin Plate DEM Generation

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Abstract

We present two new pre-processing techniques that improve thin plate Digital Elevation Model (DEM) approximations from grid-based contour data. One method computes gradients from an initial interpolated or approximated surface. The aspects are used to create gradient paths that are interpolated using Catmull-Rom splines. The computed elevations are added to the initial contour data set. Thin plate methods are applied to all of the data. The splines allow information to flow across contours, improving the final surface. The second method successively computes new, intermediate contours in between existing isolines, which provide additional data for subsequent thin plate processing. Both methods alleviate artifacts visible in previous thin plate methods. The surfaces are tested with published methods to show qualitative and quantitative improvements over previous methods.

1 Introduction

Digital Elevation Models (DEM) are often used to store three-dimensional elevation data via a regular grid. Because DEMs are not available for many areas and/or because they are storage intensive, they are often interpolated or approximated from sparse data. We have chosen isoline data from which to compute DEMs because contour maps are readily available for many geographic locations in the form of Digital Line Graphs (DLG), a standard of the United States Geological Survey (USGS). We use a grid-based approach because such methods often produce DEMs that preserve terrain morphology better than other methods, such as those using a Triangulated Irregular Network (TIN)[4]. Examples of systems that generate DEMs from contours are TOPOGRID [2][3], available in ArcInfo, TAPES-C, and TOPOG.

2 Thin Plate Splines

The notion of minimizing a thin plate to interpolate or approximate a surface is an old (i.e., [1]) and trusted technique. Given N data points, where $i \in \{1..N\}$, the differential equation that models a thin plate is given by:

$$f_i = \int\limits_R \int \left(f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2 \right) dx\ dy \tag{1}$$

where f_i is the force at position i.

Although the thin plate equation has been used in the surface reconstruction problem, one problem manifests itself more often when using contour line data as opposed to scattered data. In simplest terms, a common solution to the thin plate equation at a particular node can be stated as the weighted average of the node's neighbors. Consider contour data depicting hilly or mountainous terrain. Furthermore, consider a contour line A with a certain elevation, and a second contour line B which is at the next higher elevation (see top of Figure 1).

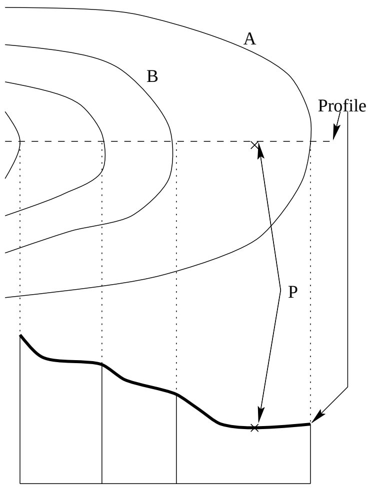

Figure 1: Profile (dark line) showing terracing problem in between contours.

3 Improving Thin Plate Methods

Our approach to improving any of the thin plate methods is to add more, accurate elevation points into the initial contour data set, without the need for operator intervention (such as adding additional peak elevation data points). This results in a two-stage method where the contours are processed first, creating a richer data set. We create additional data through the use of gradient paths and intermediate contours. The second stage applies any of the thin plate methods to yield the final surface.

The algorithm for finding all of the gradient paths in a grid of contours is as follows:

```
Compute initial thin plate surface
Compute gradient at each grid point
For each point $P_{i,j}$ on the grid not visited
    create empty path
    repeat
        mark $P_{i,j}$ as visited
        add $P_{i,j}$ to path
        if $P_{i,j}$ contains a valid gradient direction
            move to neighboring point $P_{k,l}$ following gradient direction
    until there is no valid neighbor
    apply Catmull-Rom spline to path, using contour elevations as knots
    copy new computed elevations from path back to grid
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4 Results

4.1 Evaluation criteria

The criteria used to assess the quality of a computed DEM are as follows:

1. The total squared curvature must be as low as possible. Although natural surfaces exhibit some curvature, artifacts such as the aforementioned Gibbs' phenomena contribute greatly to the total curvature. For $N = n^2$ total points, this is found by comparing each computed elevation value to its four neighbors [1]:

$$C_{sq} = \sum_{i=2}^{n-1} \sum_{j=2}^{n-1} (u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j})^2 \tag{2}$$

   where each u represents the elevation at the grid location indexed by i and j. The lower the squared curvature, the smoother the surface. This is useful for direct comparisons of results from different algorithms working on the same data.

2. DEM elevations falling on the original contour lines must have values equal to (interpolation) or almost equal to (approximation) the contour labels. This is measured by the root mean square error ($RMSE$) of the surface [5]:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (u_i - w_i)^2} \tag{3}$$

   where u_i = the interpolated DEM elevation of test point i
   w_i = the true elevation of test point i

4.2 Tests

The test case is shown in Figure 2, taken from a USGS DLG of Crater Lake, Oregon. The contours were rasterized into a 900 × 900 grid. Elevations are given in feet and the grid spacing is in meters; the contour interval is 40 feet. As is evident in the contours, the Crater Lake data has both steep sections (rising from the lake in the lower left) and flatter sections, yielding a good test for reconstruction techniques.

The results of the quantitative tests two through four are shown in Table 1. The thin plate with springs approximation yields a total squared curvature of 72,678, which indicates a globally smooth surface in relation to the other surfaces. The average curvature is among the highest, however, suggesting that there must be large areas of high curvature which may be evidence of Gibbs' phenomena. The $RMSE$ of 1.29 is 3.2% of the contour interval, which falls within the standard of five per cent. Adding tension to the thin plate creates a true interpolation ($RMSE = 0$), but at the cost of overall curvature. Visually, the gradient paths method produced a less-terraced surface, but with a few artifacts.

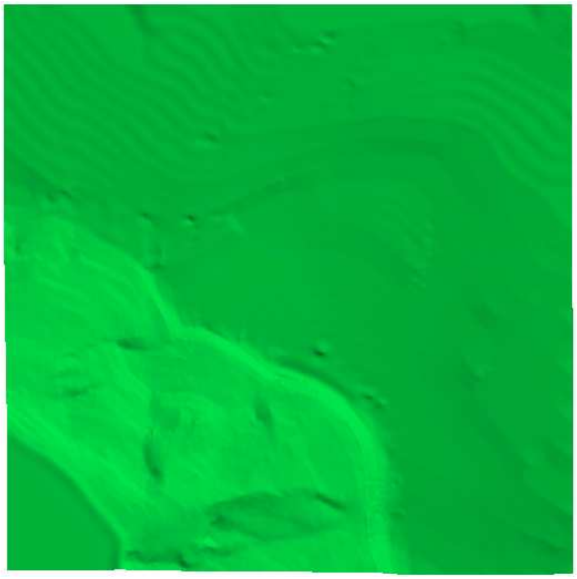

Figure 2: Crater Lake DEM obtained from intermediate contours and thin plate approximation

Table 1: Results of applying methods to Crater Lake data.

Method	C_{sq}	C_{ave}	$RMSE$	% of contour interval
Thin plate approximation	72678	0.138	1.29	3.2
Thin plate under tension interpolation	741850	0.139	0.00	0.0
TOPOGRID	128100	0.144	3.62	9.1
Gradient paths	92800	0.117	1.29	3.2
Intermediate contours	92788	0.118	1.93	4.8

Finally, to evaluate criteria six, plots were made of the relative heights of the DEMs of Crater Lake produced by each of the procedures. The plot can be seen in Figure 3. The frequency of the first height class for thin plate approximation is actually 235479; similarly, the frequency of the first height class for the thin plate under tension is 320682. Both of these indicate that the surfaces change rapidly right at the contour lines. The overall pattern of the graphs shows the terracing effect. The TOPOGRID procedure shows a very regular pattern.

5 Conclusions

The thin plate interpolation, approximation, or the addition of tension may compute smooth DEMs from contour input, but often Gibbs' phenomena and especially terracing effects are visible. The problem worsens as contour spacing increases, and is readily apparent in shaded relief maps. Automatically adding additional data in a

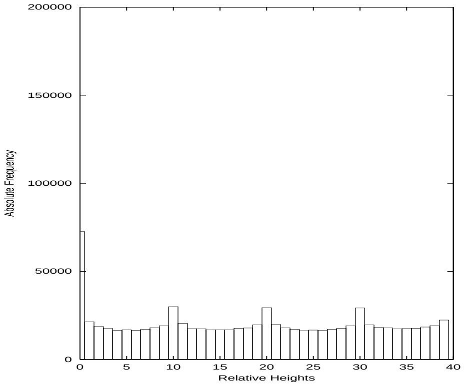

Figure 3: Thin plate with intermediate contours.

pre-processing step through either gradient paths or intermediate contours visually improve the surface created by subsequent thin plate processing, compared to thin plate methods alone. In all cases, the new methods produce smooth surfaces as shown by the total squared curvature and average curvature, while still being faithful to the original contour data, as measured by the $RMSE$. The profiles and height class plots show that the new methods create better surfaces in between contours than previous thin plate procedures alone. The surfaces compare favorably to ArcInfo's TOPOGRID procedure, the latter of which exhibited a much higher $RMSE$.

References

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